Distributed-Delay-Dependent Stabilization for Networked Interval Type-2 Fuzzy Systems With Stochastic Delay and Actuator Saturation

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***Abstract*—In this article, a distributed-delay-dependent method is proposed to deal with the stabilization for interval type-2 Takagi–Sugeno fuzzy systems subject to stochastic network delay and actuator saturation. To deal with the stochastic feature of network-induced delays, their probability density function is utilized to establish the distributed delay model, which is more practical than the traditional time-varying network delay model. Meanwhile, in order to enlarge the estimation of the domain of attraction, a polytopic strategy composed of a state vector and a distributed-delay-dependent vector is proposed to treat the saturation nonlinearity. By constructing a distributed-delay-dependent Lyapunov–Krasovskii functional, new and less conservative conditions are achieved to guarantee the stability of the established closed-loop system. Additionally, two simulation examples are carried out to illustrate the advantages of the proposed strategy.**

***Index Terms*—Actuator saturation, distributed delay, interval type-2 (IT2) fuzzy systems, networked systems, stochastic delays.**

I. Introduction

BY REPRESENTING complex nonlinear dynamics of practical systems via a series of linear subsystems accompanied with membership functions (MFs), the Takagi–Sugeno (T–S) fuzzy system has been viewed as an effective and simple way to analyze nonlinear systems. With the help of the T–S fuzzy method, a great deal of methodologies and technologies for linear systems can be applied to nonlinear systems, and many results about unmanned vehicles [1], robots [2], [3], wind turbine systems [4], [5], and circuit systems [6] can be found. Compared with the traditional type-1 T–S fuzzy system with deterministic MFs, a more general interval type-2 (IT2) T–S fuzzy system has attracted much attention and interest. By introducing the information of lower MFs (LMFs) and upper MFs (UMFs), it is able to cope with the difficulties induced by uncertain parameters in MFs, and, thus, is well studied in [7], [8], [9], [10], [11], and [12]. To be specific, a practical single-link rigid robot is described by an IT2 T–S fuzzy system in [8], where the system uncertainties are captured by utilizing LMFs and UMFs. For nonlinear mass–spring–damping system subject to cyber-attacks, the asynchronous adaptive event-triggered control problem and the sliding-mode control problem are investigated in [9] and [10], respectively, where an IT2 T–S fuzzy system used to model the uncertain parameter. By utilizing the IT2 T–S fuzzy system modeling approach to describe a one-link manipulator with the motor, an event-triggered controller under deception attacks is designed in [11] and the sampled-data H_∞ control strategy is addressed in [12]. In recent years, with the utilization of network technique, the automation degree and smart sense of practical systems, such as smart grids and unmanned vehicles, have been significantly improved [13], [14], [15], [16]. By efficiently utilizing the structure characteristic of system parameters and the sparseness of the subsystem connection matrix, the stabilization issue and the finite-time control issue for networked dynamical systems are studied in [17] and [18], respectively. For networked nonlinear systems, a distributed IT2 fuzzy load frequency control strategy is addressed for networked power systems subject to DoS attacks [19].

As the other side of the coin, the stochastic transmission delay of the signals transmitted over a wired/wireless network is an inevitable and significant issue. For practical systems, if the stochastic delay is not handled appropriately, it could deteriorate the system stability, especially, in control input where the delay is taken to build up the required control force. It is noteworthy that the conservatism of system design is decreased along with the increase of utilized delay information [20]. The simplest and extensively used method to deal with the stochastic delays is interval time-varying (ITV) delay model [21], which only utilizes the bounded delay information. This delay

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model is also applied to study the control problems for networked IT2 T–S fuzzy systems [22], [23], [24]. Compared with this model, a delay-distribution-dependent model is studied for networked control of power systems [25] and networked filtering of suspension vehicle systems [26]. Then, less conservative results are obtained due to the introduction of the distribution for short and long delay intervals, which is represented via a Bernoulli variable. Actually, for some communication networks based on Internet Protocol (IP) and IEEE 802.11, the probability density function (PDF) of random delays can be obtained by the statistical method [27], [28]. It can be viewed as the limit case of the delay-distribution-dependent model [25], [26] with infinite delay intervals. In order to use the delay distribution more accurately, however, the method in [25] and [26] is infeasible to handle the stability analysis and synthesis when the random delays are divided into infinite intervals. Thus, how to directly take into account the PDF of stochastic delays to derive less conservative results for IT2 T–S fuzzy systems is the first challenging problem and motivates this work.

In addition, for practical control systems, the input saturation of the actuator is a common phenomenon because of the physical limitations on operation range, and its existence may degrade the control performance and even result in system instability [29], [30]. Regarding this topic, some interesting and meaningful results about active suspension systems and aircraft systems are obtained in [31], [32], and [33]. There are two classical strategies to treat the nonlinearity of saturation. The one is to convert it as a local sector-bound condition [34], [35] and the other is to handle it via polytopic representation [36], [37]. Particularly, it is shown by [36] that the polytopic method can derive less conservative results than the approach in [38] for discrete-time linear systems. Following the polytopic method, Saifia et al. [39] addressed the robust H_∞ static output control of T–S fuzzy systems subject to actuator saturation. For systems with saturation and time delay simultaneously, an extra delay-dependent term is combined with the state feedback to represent the actuator saturation in [40]. With the help of the delay-dependent term, the larger estimated domain of attraction (DOA) can be derived by the approach in [40] than the way in [40] without considering it. Different from the strategy in [40] dependent on discrete time delay, a distributed-delay-dependent polytopic method is developed in [41], which is the potential to further reduce the design conservatism. In [42], the saturated impulsive control issue of nonlinear time-delay system is studied via a distributed-delay-dependent polytopic method. Nevertheless, the discrete/distributed-delay-dependent polytopic strategies in [40], [41], and [42] are infeasible to deal with the saturated system subject to random delays with PDF. Therefore, providing an effective way to design the saturated controller for IT2 T–S fuzzy systems under random delays with PDF is the second challenging issue and further inspires the current study.

According to the above observations, this article investigates the stabilization issue of networked IT2 T–S fuzzy systems subject to stochastic delays and actuator saturation. The main contributions are concluded as follows.

1) A new PDF-based distributed delay model is presented to describe the network-induced stochastic delays. Compared with the traditional ITV delay model in [12], the more specific information of stochastic delay, PDF, is utilized and treated as the distributed delay kernel in our proposed method.
2) A novel polytopic distributed-delay-dependent method is proposed to handle the nonlinearity of actuator saturation. Different from the existing polytopic method in [39] only based on the system state, an extra term with respect to the PDF-based distributed delay is considered in this work, which can reduce design conservatism and obtain a larger estimation value of DOA.
3) A constructed distributed-delay-dependent Lyapunov–Krasovskii functional (LKF) and an integral inequality related to the PDF are used to derive new T–S fuzzy stabilization conditions. In contrast to the existing method using Legendre polynomials to approximate the kernel in [43], the distributed delay kernel is handled directly without introducing any approximation error, which means this strategy can lead to the less conservative conditions than the method in [43].

The outline of this work is presented as follows. The preliminaries of modeling the saturated control system with stochastic network delay are provided in Section II. In Section III, the stability and controller design conditions are derived. Then, the effectiveness of the proposed method is verified via some simulations in Section IV. Section V summarizes the conclusions and gives some future research.

Notation: In this article, the 2-norm and ∞-norm are denoted by $\|\cdot\|_2$ and $\|\cdot\|_\infty$, respectively. $A^\top$ denotes the transpose of a matrix or vector A. $\Im(A, B)$ and $\mathrm{He}(A)$ represent $B^\top AB$ and $A^\top + A$, respectively. $\mathbb{I}[1, a]$ means the set $\{1, \ldots, a\}$. $\otimes$ stands for Kronecker product. $\mathrm{eig}_m(A)$ denotes the maximum eigenvalue of matrix A.

II. Preliminaries

The considered IT2 T–S fuzzy dynamic system is represented by IF-THEN rules as follows.

Plant Rule i: **IF** $\theta_1(x(t))$ is $\mathscr{X}_1^i$ and ... and $\theta_r(x(t))$ is $\mathscr{X}_r^i$, **THEN**

$$\dot{x}(t) = A_i x(t) + B_i \mathbb{S}(u(t)) \tag{1}$$

where $\mathscr{X}_r^i (i = 1, \ldots, p)$ is the fuzzy set, $\theta(x(t)) = [\theta_1(x(t)), \ldots, \theta_r(x(t))]^T$ denotes the vector of premise variables, $x(t) \in \mathbb{R}^n$ means the state, $u(t) \in \mathbb{R}^g$ means the control input, the nonlinear saturation function with unity level $\mathbb{S}(u(t))$ is represented as $\mathbb{S}(u(t)) = \begin{bmatrix} \mathbb{S}(u_1(t)) & \cdots & \mathbb{S}(u_f(t)) & \cdots & \mathbb{S}(u_g(t)) \end{bmatrix}^\top$, $\mathbb{S}(u_f(t)) = sgn(u_f(t)) \min\{1, |\bar{u}_f|\}$, and the matrices A_i and B_i are system matrices.

The firing strength of the ith rule is defined as

$$\Theta_i(\theta(t)) = \begin{bmatrix} \underline{\vartheta}_i(\theta(t)) & \overline{\vartheta}_i(\theta(t)) \end{bmatrix} \tag{2}$$

where

$$\underline{\vartheta}_i(\theta(t)) = \prod_{m=1}^{r} \underline{\eta}_{\mathscr{X}_m^i}(\theta_m(t)) \geq 0$$

$$\overline{\vartheta}_i(\theta(t)) = \prod_{m=1}^{r} \overline{\eta}_{\mathscr{X}_m^i}(\theta_m(t)) \geq 0$$

$$\overline{\vartheta}_i(\theta(t)) \geq \underline{\mu}_i(\theta(t)) \geq 0$$
$$\overline{\eta}_{\mathscr{X}_m^i}(\theta_m(t)) \geq \underline{\eta}_{\mathscr{X}_m^i}(\theta_m(t)) \geq 0$$

with $\underline{\vartheta}_i(\theta(t))$ and $\overline{\vartheta}_i(\theta(t))$ denoting LMF and UMF, respectively; and $\underline{\eta}_{\mathscr{X}_m^i}(\theta_m(t))$ and $\overline{\eta}_{\mathscr{X}_m^i}(\theta_m(t))$ representing the grades of membership of $\theta_m(t)$ in LMF and UMF, respectively.

Then, the defuzzified system is captured by

$$\dot{x}(t) = \sum_{i=1}^{p} \vartheta_i(\theta(t))[A_i x(t) + B_i \mathbb{S}(u(t))] \tag{3}$$

where

$$\vartheta_i(\theta(t)) = \frac{\mu_i(\theta(t))}{\sum_{i=1}^{r} \mu_i(\theta(t))}$$
$$\mu_i(\theta(t)) = \underline{\beta}_i(\theta(t))\underline{\vartheta}_i(\theta(t)) + \overline{\beta}_i(\theta(t))\overline{\vartheta}_i(\theta(t))$$
$$0 \leq \underline{\beta}_i(\theta(t)) \leq 1,\ \ 0 \leq \overline{\beta}_i(\theta(t)) \leq 1$$
$$\underline{\beta}_i(\theta(t)) + \overline{\beta}_i(\theta(t)) = 1$$

with $\vartheta_i(\theta(t))$ being the normalized MF, and $\underline{\beta}_i(\theta(t))$ and $\overline{\beta}_i(\theta(t))$ being the nonlinear weighting functions of the ith rule.

The stochastic transmission delays are considered when the control signals are communicated over the network channel. Then, the control law of the jth ($j \in \mathscr{P}$) rule is presented as follows. *Controller Rule j*: **IF** $\theta_1(x(t))$ is $\mathscr{X}_1^j$ and ... and $\theta_r(x(t))$ is $\mathscr{X}_r^j$, **THEN**

$$u(t) = K_j x(t - \tau(t)) \tag{4}$$

in which K_j and $\tau(t) \in [0,\ \ h]$ denote the controller gain and the stochastic transmission delay, respectively.

In a practical network environment, the stochastic transmission delay $\tau(t)$ commonly satisfies some probability distributions. To utilize the probability information of $\tau(t)$, its PDF $m(\cdot)$ is considered to approximate the probability distribution. By applying the same way to obtain system (3) and the PDF of stochastic network-induced delay, a defuzzified distributed-delay-dependent controller is expressed as

$$u(t) = \sum_{j=1}^{p} \vartheta_j(t - \tau(t)) K_j \int_{-h}^{0} m(s)x(t+s)ds \tag{5}$$

where the kernel $m(s)$ represents the PDF and $\int_{-h}^{0} m(s)ds = 1$. In the following derivation, $\vartheta_i(t)$ and $\vartheta_j(t - \tau(t))$ are abbreviated as ϑ_i and ϑ_j^{τ}, respectively. Furthermore, the condition $\vartheta_j^{\tau} - \nu_j \vartheta_j \geq 0$ is supposed to be satisfied.

Remark 1: For networked IT2 fuzzy systems with stochastic delays, the stochastic distribution information, PDF ($m(s)$), is taken into account in the proposed distributed delay model $\int_{-h}^{0} m(s)x(t+s)ds$. Compared with the traditional ITV delay model $x(t - \tau(t))$ in [12] to handle the stochastic delays, our distributed delay model is more general and practical, which is useful for deriving less conservative results.

Lemma 1 [36]: For given integer $g \geq 1$ and function $\gamma(t) \in \mathbb{R}^{\overleftrightarrow{g}}$ satisfying $\|\gamma(t)\|_\infty \leq 1$, $\overleftrightarrow{g} = g2^{g-1}$, the function w_g is defined as $w_g(0) = 0$ and

$$w_g(r) = \begin{cases} w_g(r-1) + 1, & E_r + E_j \neq I_g\ \forall j \in \mathbb{I}[1, r] \\ w_g(j), E_r + E_j = I_g, & \exists j \in \mathbb{I}[1, r] \end{cases}$$

there exists

$$\mathbb{S}(u(t)) \in \complement\left\{E_r u(t) + \mathcal{E}_r^- \gamma(t) : r \in \mathbb{I}\left[1, 2^g\right]\right\} \tag{6}$$

holds for any $u(t) \in \mathbb{R}^g$, where $\complement$ means the convex hull, $E_r^- = I - E_r$, $\mathcal{E}_r^- \triangleq e_{2^{g-1}, w_g(r)} \otimes E_r^- \in \mathbb{R}^{g \times \overleftrightarrow{g}}$, $e_{2^{g-1}, w_g(r)}$ means a row vector and its elements are 0 except for the $w_g(r)$th element is 1.

We assume that there exist matrices $U, F \in \mathbb{R}^{\overleftrightarrow{g} \times n}$ such that

$$\|\gamma(t)\|_\infty = \left\| Ux(t) + F \int_{-h}^{0} m(s)x(t+s)ds \right\|_\infty \leq 1. \tag{7}$$

According to (6), the nonlinear saturation function $\mathbb{S}(u(t))$ can be expressed as

$$\mathbb{S}(u(t)) = \sum_{r=1}^{2^g} \varsigma_r^t \left[E_r u(t) + \mathcal{E}_r^- \gamma(t) \right] \tag{8}$$

where $\varsigma_r^t \geq 0$ and $\sum_{r=1}^{2^g} \varsigma_r^t = 1$.

Remark 2: The distributed-delay-dependent information $\int_{-h}^{0} m(s)x(t+s)ds$ induced by stochastic transmission delay is used to represent the saturation nonlinearity (8). Compared with the conventional method only based on the state $x(t)$ in [36], [37], and [39], the PDF-based distributed delay is involved for the first time in this article, which is helpful in decreasing the conservatism.

Remark 3: In [41], an existing polytopic distributed-delay saturation representation method is studied as

$$\|\gamma(t)\|_\infty = \left\| Ux(t) + \sum_{l=1}^{N} F_l \int_{-ld}^{-(l-1)d} x(t+s)ds \right\|_\infty \leq 1 \tag{9}$$

where $Nd = h$. If $F_l = Fm_l$ and $\int_{-ll}^{-(l-1)d} m(s)ds = m_l$ are chosen, then we have

$$\|\gamma(t)\|_\infty = \left\| Ux(t) + \sum_{l=1}^{N} F \int_{-ld}^{-(l-1)d} m_l x(t+s)ds \right\|_\infty \leq 1 \tag{10}$$

the limitation of which for $N \to \infty$ equals to (7). In this case, it could cause infinite dimensions of analysis conditions by utilizing the strategy in [41]. Thus, it is infeasible to solve the saturated control issue for the distributed-kernel-based delay system (1).

By combining (3), (7), and (8), the closed-loop system is deduced as

$$\dot{x}(t) = \sum_{r=1}^{2^g} \varsigma_r^t \left[\sum_{i=1}^{p} \sum_{j=1}^{p} \vartheta_i \vartheta_j^{\tau} \left(\left(A_i + B_i \mathcal{E}_r^- U\right)x(t) + \left(B_i E_r K_j + B_i \mathcal{E}_r^- F\right) \int_{-h}^{0} m(s)x(t+s)ds \right) \right]. \tag{11}$$

In order to deal with the distributed delay item $\int_{-h}^{0} m(s) x(t+s)ds$, we define $m(s) = m_0(s)$ and

$$\mathbf{m}(s) = \left[m_0(s)\ \ m_1(s)\ \ \cdots\ \ m_\kappa(s)\right]^\top$$
$$M(s) = \mathbf{m}(s) \otimes I_n,\ \ \mathscr{I} = \left[I_n\ \ 0_{n \times \kappa n}\right]$$
$$\mathbb{M}(t) = \int_{-h}^{0} M(s)x(t+s)ds. \tag{12}$$

In terms of [44], the basic principle to select $m_i(s)$ is that it should make the vector $\mathbf{m}(s)$ satisfy the property

$$\dot{\mathbf{m}}(s) = \mathcal{M}\mathbf{m}(s) \tag{13}$$

which means that the elements $m_i(s)$ are the solutions of linear homogeneous differential equations with constant coefficients in the matrix $\mathcal{M} \in \mathbb{R}^{n(\kappa+1)\times n(\kappa+1)}$.

Then, the closed-loop networked IT2 T–S fuzzy system is rewritten as

$$\dot{x}(t) = \sum_{r=1}^{2^g} \varsigma_r^t \Bigg[\sum_{i=1}^{p}\sum_{j=1}^{p} \vartheta_i \vartheta_j^\tau \left(\left(A_i + B_i \mathcal{E}_r^- U\right) x(t) + \left(B_i E_r K_j + B_i \mathcal{E}_r^- F\right) \mathscr{I} \mathbb{M}(t) \right) \Bigg]. \tag{14}$$

Before ending this section, a technical lemma is provided as follows.

Lemma 2 [44]: For a matrix $J > 0 \in \mathbb{R}^{n\times n}$, $J = J^\top$ and the vector $\mathbf{m}(s)$ defined in (12), it yields

$$\int_{-h}^{0} \Im(J, x(s))ds \geq \Im\left(\mathbf{M}\otimes J, \int_{-h}^{0} M(s)x(s)ds\right) \tag{15}$$

with $\mathbf{M}^{-1} = \int_{-h}^{0} \mathbf{m}(s)\mathbf{m}^\top(s)ds > 0$.

III. Main Results

For the simplicity of analysis and derivation, the following abbreviation is defined as:

$$\mathbf{e}_l \triangleq \left[0_{n,n(l-1)}\ I_n\ 0_{n,n(4+\kappa-l)}\right],\ l = 1, \ldots, 4+\kappa.$$

First, the stability analysis conditions for system (14) with stochastic delays and input saturation are formed in the next theorem.

Theorem 1: For given constants α, ν_i, h and any initial condition satisfying $\mathcal{V}(0) \leq 1$, under the controller gain K_j, the system (14) is asymptotically stable, if there exist symmetric matrices $P > 0$, $W > 0$, and $Y > 0$, and matrices $\Re_i$, X, U, and F such that

$$\Psi_{ij}^r - \Re_i < 0 \tag{16}$$

$$\Upsilon_{ii}^r < 0 \tag{17}$$

$$\Upsilon_{ij}^r + \Upsilon_{ji}^r < 0, \quad (i < j) \tag{18}$$

$$\begin{bmatrix} 1 & H_q^\top \\ H_q & \mathscr{P} \end{bmatrix} \geq 0 \tag{19}$$

where

$$\Psi_{ij}^r = \Omega + \mathrm{He}\left(\mathbf{X}\mathbf{S}_{ij}^r\right),\ \Upsilon_{ij}^r = \nu_j\left(\Psi_{ij}^r - \Re_i\right) + \Re_i$$

$$\Omega = \mathrm{He}\left(\Phi_1^\top P \Phi_2\right) + \Im(W + hY, \mathbf{e}_2) - \Im(W, \mathbf{e}_3) - \Im(\mathscr{Y}, \mathbf{e}_a)$$

$$\Phi_1 = \begin{bmatrix} \mathbf{e}_2 \\ \mathbf{e}_a \end{bmatrix}, \Phi_2 = \begin{bmatrix} \mathbf{e}_1 \\ M(0)\mathbf{e}_2 - M(-h)\mathbf{e}_3 - \mathcal{M}\mathbf{e}_a \end{bmatrix}, \mathbf{e}_a = \begin{bmatrix} \mathbf{e}_4 \\ \vdots \\ \mathbf{e}_{4+\kappa} \end{bmatrix}$$

$$\mathbf{X} = \mathbf{e}_1^\top X + \alpha \mathbf{e}_2^\top X,\ \mathscr{Y} = \mathbf{M}\otimes Y$$

$$\mathbf{S}_{ij}^r = -\mathbf{e}_1 + \left(A_i + B_i\mathcal{E}_r^- U\right)\mathbf{e}_2 + \left(B_i E_r K_j + B_i \mathcal{E}_r^- F\right)\mathscr{I}\mathbf{e}_a$$

$$\mathscr{P} = P + \mathrm{diag}\{0, \mathscr{W}\},\ \mathscr{W} = \mathbf{M}\otimes W,\ H_q = \left[U_q\ \ F_q\mathscr{I}\right].$$

Proof: Define $\varphi(t) = \begin{bmatrix} x(t) \\ \mathbb{M}(t) \end{bmatrix}$ and construct an LKF as

$$\mathcal{V}(t) = \mathcal{V}_1(t) + \mathcal{V}_2(t) \tag{20}$$

where

$$\mathcal{V}_1(t) = \Im(P, \varphi(t))$$

$$\mathcal{V}_2(t) = \int_{t-h}^{t} \Im(W + (s-t+h)Y, x(s))ds.$$

By differentiating $\mathcal{V}(t)$, it gives

$$\dot{\mathcal{V}}(t) = \mathrm{He}\left(\varphi^\top(t)P\dot{\varphi}(t)\right) + \Im(W + hY, x(t)) - \Im(W, x(t-h)) - \int_{-h}^{0} \Im(Y, x(t+s))ds. \tag{21}$$

Applying Lemma 2 to handle the integral item, one can get

$$-\int_{-h}^{0} \Im(Y, x(t+s))ds \leq -\Im(\mathscr{Y}, \mathbb{M}(t)). \tag{22}$$

Based on the property $\dot{\mathbf{m}}(s) = \mathcal{M}\mathbf{m}(s)$, it leads to

$$\dot{\mathbb{M}}(t) = M(0)x(t) - M(-h)x(t-h) - \mathcal{M}\mathbb{M}(t). \tag{23}$$

Define

$$\eta^\top(t) = \left[\dot{x}^\top(t), x^\top(t), x^\top(t-h), \mathbb{M}^\top(t)\right]. \tag{24}$$

According to (23) and (24), it yields

$$\varphi(t) = \Phi_1\eta(t),\ \dot{\varphi}(t) = \Phi_2\eta(t). \tag{25}$$

By constructing $\mathbf{X} = X\mathbf{e}_1^\top + \alpha X\mathbf{e}_2^\top$, it is derived from system (14) that

$$\sum_{r=1}^{2^g} \varsigma_r^t \left(\sum_{i=1}^{p}\sum_{j=1}^{p} \vartheta_i\vartheta_j^\tau \Im\left(\mathbf{X}\mathbf{S}_{ij}^r, \eta(t)\right)\right) = 0. \tag{26}$$

To guarantee the asymptotic stability of the closed-loop system (14), one needs

$$\begin{aligned}\dot{\mathcal{V}}(t) &\leq \eta^\top(t)\mathrm{He}\left(\Phi_1^{\top P}\Phi_2\right)\eta(t) + \Im(W + hY, \mathbf{e}_2\eta(t)) \\ &\quad - \Im(W, \mathbf{e}_3\eta(t)) - \Im(\mathscr{Y}, \mathbf{e}_a\eta(t))) \\ &= \Im(\Omega, \eta(t)) < 0. \end{aligned} \tag{27}$$

By adding (26) to (27), we have

$$\sum_{r=1}^{2^g} \varsigma_r^t \left(\sum_{i=1}^{p}\sum_{j=1}^{p} \vartheta_i\vartheta_j^\tau \Im\left(\Psi_{ij}^r, \eta(t)\right)\right) < 0 \tag{28}$$

which can be ensured by

$$\sum_{i=1}^{p}\sum_{j=1}^{p} \vartheta_i\vartheta_j^\tau \Im\left(\Psi_{ij}^r, \eta(t)\right) < 0. \tag{29}$$

In terms of $\sum_{i=1}^{p} \vartheta_j = \sum_{i=1}^{p} \vartheta_j^\tau = 1$, it yields

$$\sum_{i=1}^{p}\sum_{j=1}^{p} \vartheta_i\left(\vartheta_j - \vartheta_j^\tau\right)\Im(\Re_i, \eta(t)) = 0. \tag{30}$$

Then, one deduces

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\vartheta_j^\tau\Im\big(\Psi_{ij}^r,\eta(t)\big)=\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\vartheta_j^\tau\Im\big(\Psi_{ij}^r,\eta(t)\big)+\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\Big(\vartheta_j-\vartheta_j^\tau\Big)\Im\big(\Re_i,\eta(t)\big)$$
$$=\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\vartheta_j^\tau\Im\big(\Psi_{ij}^r-\Re_i,\eta(t)\big)+\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\vartheta_j\Im\big(\Re_i,\eta(t)\big).\quad(31)$$

By considering $\vartheta_j^\tau-\nu_j\vartheta_j\geq 0$ and (16), it leads to

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\vartheta_j^\tau\Im\big(\Psi_{ij}^r,\eta(t)\big)\leq\sum_{i=j=1}^{p}\vartheta_i\vartheta_j\varsigma^\top(t)\Im\Big(\nu_j\Big(\Psi_{ij}^r-\Re_i\Big),\eta(t)\Big)+\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i\vartheta_j\Im\big(\Re_i,\eta(t)\big)$$
$$\leq\sum_{i=j=1}^{p}\vartheta_i\vartheta_j\Im\Big(\nu_i\Big(\Psi_{ij}^r-\Re_i\Big)+\Re_i,\eta(t)\Big)+\sum_{i=1}^{p}\sum_{i<j}\vartheta_i\vartheta_j\Im\Big(\nu_j\Big(\Psi_{ij}^r-\Re_i\Big)+\Re_i+\nu_i\Big(\Psi_{ij}^r-\Re_j\Big)+\Re_j,\eta(t)\Big)\quad(32)$$

which makes (28) hold by (16)–(18) and further guarantees the system stability.

In terms of (28), one obtains $\dot{\mathcal{V}}(t)<0$ and

$$\mathcal{V}(0)>\mathcal{V}(t).\quad(33)$$

From Lemma 2, the integral term $\int_{t-h}^{t}\Im(W,x(s))ds$ can be relaxed as

$$\int_{t-h}^{t}\Im(W,x(s))ds\leq\Im(\mathscr{W},\mathbb{M}(t)).\quad(34)$$

According to (34), it further results in

$$\Im(\mathscr{P},\varphi(t))\leq\Im(P,\varphi(t))+\Im(\mathscr{W},\mathbb{M}(t))+\int_{-h}^{0}\Im((s+h)Y,x(t+s))ds\leq\mathcal{V}(t).\quad(35)$$

It is assumed that

$$H_q^\top H_q\leq\mathscr{P},\quad q\in\mathbb{I}\big[1,\ \overleftrightarrow{g}\big]\quad(36)$$

which is equivalent to

$$\begin{bmatrix}1 & H_q^\top\\ H_q & \mathscr{P}\end{bmatrix}\geq 0\quad(37)$$

where $H_q=\big[U_q\ \ F_q\mathscr{I}\big]$, H_q is the qth row of H.

According to (36) and (37), one gets

$$\big|U_qx(t)+F_q\mathscr{I}\mathbb{M}(t)\big|^2=\Im\Big(H_q^\top H_q,\varphi(t)\Big)\leq\Im(\mathscr{P},\varphi(t)).\quad(38)$$

For any initial condition meeting $\mathcal{V}(0)\leq 1$, it is observed from (33), (35), and (38) that $|U_qx(t)+F_q\mathscr{I}\mathbb{M}(t)|^2\leq 1$ holds. This indicates that the assumption (7) is ensured.

Therefore, the asymptotic stability of the closed-loop system (14) is ensured for any initial state satisfying $\mathcal{V}(0)\leq 1$. ■

Remark 4: In [43], Legendre polynomials are employed to treat the PDF $m(s)$, which will cause an approximation error. Compared to this approach, this article adopts integral inequality in Lemma 2 to deal with the PDF directly, and no approximation error is introduced.

Second, according to the results in Theorem 1, the fuzzy saturated controller design conditions formulated by linear matrix inequalities (LMIs) are deduced in Theorem 2.

Theorem 2: For given scalars α, ν_i, and h, if there exist symmetric matrices $\check{P}>0$, $\check{W}>0$, and $\check{Y}>0$, and matrices $\check{\Re}_i$, Q, Z_U, and Z_F such that for $\forall r\in\mathbb{I}[1,2^g]$ $\forall q\in\mathbb{I}[1,\ \overleftrightarrow{g}]$, the following LMIs hold:

$$\check{\Psi}_{ij}^r-\check{\Re}_i<0\quad(39)$$
$$\check{\Upsilon}_{ii}^r<0\quad(40)$$
$$\check{\Upsilon}_{ij}^r+\check{\Upsilon}_{ji}^r<0,\quad(i<j)\quad(41)$$
$$\begin{bmatrix}1 & \check{H}_q^\top\\ \check{H}_q & \check{\mathscr{P}}\end{bmatrix}\geq 0\quad(42)$$

where

$$\check{\Psi}_{ij}^r=\check{\Omega}+\mathrm{He}\Big(\hat{\mathbf{X}}\check{\mathbf{S}}_{ij}^r\Big),\ \check{\Upsilon}_{ij}^r=\nu_j\Big(\check{\Psi}_{ij}^r-\check{\Re}_i\Big)+\check{\Re}_i$$
$$\check{\Omega}=\mathrm{He}\Big(\Phi_1^\top\check{P}\Phi_2\Big)+\Im\Big(\check{W}+h\check{Y},\mathbf{e}_2\Big)-\Im\Big(\check{W},\mathbf{e}_3\Big)-\Im\Big(\check{\mathscr{Y}},\mathbf{e}_a\Big)$$
$$\check{\mathbf{X}}=\mathbf{e}_1^\top+\alpha\mathbf{e}_2^\top,\ \check{\mathscr{Y}}=\mathbf{M}\otimes\check{Y}$$
$$\check{\mathbf{S}}_{ij}^r=\big(A_iQ+B_i\mathcal{D}_r^-Z_U\big)\mathbf{e}_2-Q\mathbf{e}_1+\big(B_iD_rZ_{Kj}+B_i\mathcal{D}_r^-Z_F\big)\mathscr{I}\mathbf{e}_a$$
$$\check{\mathscr{P}}=\check{P}+\mathrm{diag}\Big\{0,\check{\mathscr{W}}\Big\},\ \check{\mathscr{W}}=\mathbf{M}\otimes\check{W},\ \check{H}_q=\big[Z_{Uq}\ \ Z_{Fq}\mathscr{I}\big].$$

Then for any initial condition satisfying $\mathcal{V}(0)\leq 1$, the asymptotic stability of system (14) can be ensured by the controller $K_j=Z_{Kj}Q^{-\top}$.

Proof: Define $Q=X^{-1}$, $\check{\Re}_i=\Im(\Re_i,I_{n(\kappa+4)}\otimes Q^\top)$, $\check{W}=\Im(W,Q^\top)$, $\check{Y}=\Im(Y,Q^\top)$, $\check{\mathscr{Y}}=\Im(\mathscr{Y},I_{(\kappa+1)}\otimes Q^\top)$, $\check{\mathscr{W}}=\Im(\mathscr{W},I_{(\kappa+1)}\otimes Q^\top)$, $K_jQ^\top=Z_{Kj}$, and $H_q(I_{(\kappa+2)}\otimes Q^\top)=\big[Z_{Uq}\ Z_{Fq}\mathscr{I}\big]$.

Pre- and post-multiplying (16) with $\mathscr{Q}=I_{n(\kappa+4)}\otimes Q$ and $\mathscr{Q}^\top$, we get

$$\check{\Psi}_{ij}-\check{\Re}_i<0\quad(43)$$

which equals to (39).

Following a similar way in the above, the conditions (40)–(42) are also obtained, which completes the proof. ■

Remark 5: If the saturation level is nonunity with $\mathbb{S}(u_j)=sgn(u_j)\min\{|u_j|,\bar{u}_j\}$, the matrices B and Z_{Kj} in Theorem 2 should be replaced by $\bar{B}=[\bar{u}_1b_1\ \bar{u}_2b_2\ \cdots\ \bar{u}_gb_g]$ and $\bar{Z}_{Kj}=[z_{Kj1}^\top/\bar{u}_1\ z_{Kj2}^\top/\bar{u}_2\ \cdots\ z_{Kjg}^\top/\bar{u}_g]^\top$, respectively, where b_f is the fth column of B and $z_{Kjf}^\top$ is the fth row of Z_{Kj}.

Remark 6: The computational complexity of the proposed approach is related to the number of matrix inequality conditions ($\mathscr{N}$) in Theorems 1 and 2. It is computed as $\mathscr{N}=2^p\cdot 2^g+p\cdot 2^g+([p(p-1)]/2)\cdot 2^g+g\cdot 2^{g-1}$, where g and p

denote the number of control channels and fuzzy rules. From the above equation, one observes that the growth of g and p could result in the exponential increasement of the computational complexity of the developed approach, which may decrease its feasibility in some practical multichannel systems.

Next, an optimization problem is proposed to derive a larger estimation of DOA ($\mathbb{A}_\delta$) when the controller is designed. According to the chosen LKF (20), we have

$$\begin{aligned}\mathcal{V}_1(t) &\le \Im(P, \varphi(t)) \le \Im(\text{diag}\{\Lambda_0, \Lambda_1\}, \varphi(t)) \\ &\le \Im\big(\text{eig}_m(\Lambda_0), x(t)\big) + \Im\Big(\big(\text{eig}_m(\Lambda_1), \mathbb{M}(t)\Big) \\ &\le \Big(\text{eig}_m(\Lambda_0) + h\text{eig}_m(\Lambda_1)\text{eig}_m\Big(\mathbf{M}^{-1}\Big)\Big)\|x(t)\|_2^2 \quad (44)\end{aligned}$$

$$\mathcal{V}_2(t) \le \Big(h\text{eig}_m(W) + h^2\text{eig}_m(Y)\Big)\|x(t)\|_2^2. \quad (45)$$

Then, the bound of $\mathbb{A}_\delta$ can be estimated by

$$\begin{aligned}\mathcal{V}(0) \le \big(&\text{eig}_m(\Lambda_0) + h\varpi\text{eig}_m(\Lambda_1) \\ &+ h\text{eig}_m(W) + h^2\text{eig}_m(Y)\big)\delta^2\end{aligned} \quad (46)$$

where $\|x(0)\|_2 \le \delta$ and $\varpi = \text{eig}_m(\mathbf{M}^{-1})$.

As in [41], the constraint $Q^{-1}Q^{-\top} \le \mu I$ is considered, which is ensured by

$$\begin{bmatrix}\mu I & I \\ I & \text{He}(Q) - I\end{bmatrix} \ge 0 \quad (47)$$

where $\mu > 0$ is a variable scalar.

Define

$$\begin{aligned}&\check{P} \le \check{\Lambda}, \ \ \check{\Lambda}_l \le \lambda_l I, \quad l = 0, 1 \\ &\check{Y} \le yI, \ \ \check{W} \le wI\end{aligned} \quad (48)$$

where $\check{\Lambda}_0 = Q^{-1}\Lambda_0 Q^{-\top}$, $\check{\Lambda}_1 = \Im(\Lambda_1, I_{(\kappa+1)} \otimes Q^{-\top})$, and $\check{\Lambda} \triangleq \text{diag}\{\check{\Lambda}_0, \check{\Lambda}_1\}$.

Then, it is observed from (46) that the maximization of $\mathbb{A}_\delta$ in Theorem 2 can be optimized by the following optimization problem.

Problem 1: $\min_{\check{P},\check{Y},\check{W},\Lambda_l,Z_{Kj},Z_U,Z_F,Q,\mu,\lambda_l,y,w}$ ρ subject to LMIs (47), (48), (39)–(42), where $\rho = \epsilon\mu + (\lambda_0 + h\varpi\lambda_1 + hw + h^2 y)$ and ϵ is the weighting parameter. Consequently, the maximum δ is derived by $\delta_{\max} = \sqrt{1/\mho}$, where

$$\mho = \text{eig}_m(\Lambda_0) + h\varpi\text{eig}_m(\Lambda_1) + h\text{eig}_m(W) + h^2\text{eig}_m(Y).$$

In addition, by setting the auxiliary matrix $F = 0$, the proposed distributed-delay-dependent method reduces to the traditional delay-independent model in [39] as

$$\mathbb{S}(u(t)) = \sum_{r=1}^{2^g} \varsigma_r^t\big(E_r u(t) + \mathcal{E}_r^- U x(t)\big). \quad (49)$$

Thus, the resulting closed-loop system is derived as

$$\begin{aligned}\dot{x}(t) = \sum_{r=1}^{2^g} \varsigma_r^t \Bigg[\sum_{i=1}^{2}\sum_{j=1}^{2} \vartheta_i \vartheta_j^\tau \Big(\big(A_i + B_i\mathcal{E}_r^- U\big)x(t) \\ + \ B_i E_r K_j \mathscr{I}\mathbb{M}(t)\Big)\Bigg].\end{aligned} \quad (50)$$

For system (50), the corresponding controller design conditions are given in Corollary 1.

Corollary 1: For given scalars α, ν_i, and h, if there exist symmetric matrices $\check{P} > 0$, $\check{W} > 0$, and $\check{Y} > 0$, and matrices $\check{\Re}_i$, Q, and Z_U such that

$$\tilde{\Psi}_{ij}^r - \check{\Re}_i < 0 \quad (51)$$

$$\tilde{\Upsilon}_{ii}^r < 0 \quad (52)$$

$$\tilde{\Upsilon}_{ij}^r + \tilde{\Upsilon}_{ji}^r < 0, \quad (i < j) \quad (53)$$

$$\begin{bmatrix}1 & \tilde{H}_g^\top \\ \tilde{H}_q & \mathscr{P}\end{bmatrix} \ge 0 \quad (54)$$

hold for $\forall r \in \mathbb{I}[1, 2^g]$ $\forall q \in \mathbb{I}[1, \overleftrightarrow{g}]$, where

$$\begin{aligned}\tilde{\Psi}_{ij}^r &= \check{\Omega} + \text{He}\Big(\check{\mathbf{X}}\tilde{\mathbf{S}}_{ij}^r\Big) \\ \tilde{\Upsilon}_{ij}^r &= \nu_j\Big(\tilde{\Psi}_{ij}^r - \check{\Re}_i\Big) + \check{\Re}_i, \ \ \tilde{H}_q = \big[Z_{Uq} \ \ 0\big] \\ \tilde{\mathbf{S}}_{ij}^r &= -Q\mathbf{e}_1 + \big(A_i Q + B_i \mathcal{E}_r^- Z_U\big)\mathbf{e}_2 + B_i E_r Z_{Kj}\mathscr{I}\mathbf{e}_a.\end{aligned}$$

Then for any initial condition satisfying $\mathcal{V}(0) \le 1$, the asymptotic stability of system (50) can be ensured by the controller $K_j = Z_{Kj}Q^{-\top}$.

Similar to the optimization issue (Problem 1), the maximization of $\mathbb{A}_\delta$ in Corollary 1 can be solved by the following.

Problem 2: $\min_{\check{P},\check{Y},\check{W},\Lambda_l,Z_{Kj},Z_U,Q,\mu,\lambda_l,y,w}$ ρ subject to LMIs (47), (48), (51)–(54), where $\rho = \epsilon\mu + (\lambda_0 + h\varpi\lambda_1 + hw + h^2 y)$ and ϵ is the weighting parameter. Correspondingly, we have $\delta_{\max} = \sqrt{1/\mho}$.

IV. Example

In this section, to show the advantages of the proposed method, two examples are executed as follows. To be specific, the comparison results in Example 1 illustrate that the presented PDF-based delay model could lead to less conservative results than the traditional ITV delay model. In addition, Example 2 demonstrates that the proposed distributed-delay-dependent method to deal with actuator saturation can reduce the design conservatism compared with the existing delay-independent method.

Example 1: Rossler's equation with control input borrowed from [45] is considered as

$$\begin{cases}\dot{x}_1(t) = -x_2(t) - x_3(t) \\ \dot{x}_2(t) = x_1(t) + a_1 x_2(t) \\ \dot{x}_3(t) = a_2 x_1(t) - (a_3(t) - x_1(t))x_3(t) + u(t)\end{cases} \quad (55)$$

where $x_1(t) \in [a_3(t) - a_4, \ a_3(t) + a_4]$, and suppose $a_1 = 0.2$, $a_2 = 1$, $a_4 = 12$, and $a_3(t) \in [0, \ 5]$.

The system (55) is formed as the fuzzy system (1) with matrices

$$A_1 = \begin{bmatrix}0 & -1 & -1 \\ 1 & a_1 & 0 \\ a_2 & 0 & -a_4\end{bmatrix}, \ A_2 = \begin{bmatrix}0 & -1 & -1 \\ 1 & a_1 & 0 \\ a_2 & 0 & a_4\end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^\top$$

and the MFs are

$$\begin{aligned}\vartheta_1(x_1(t)) &= 0.5 + \frac{a_3(t) - x_1(t)}{2a_4} \\ \vartheta_2(x_1(t)) &= 1 - \vartheta_1(x_1(t)).\end{aligned}$$

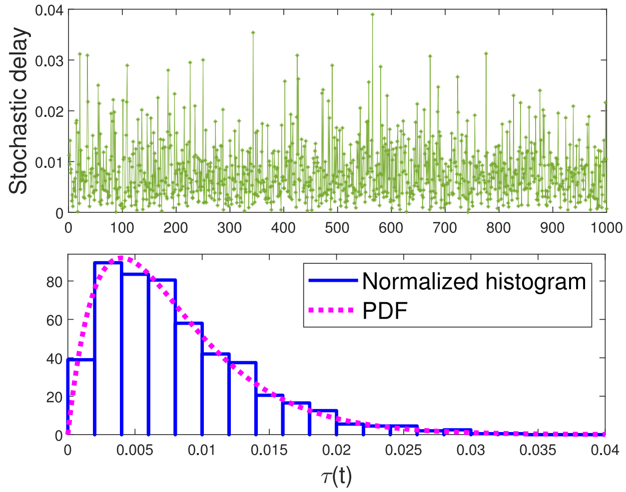

Fig. 1. Stochastic network delay and its PDF for $h = 0.04$ s.

Since the MFs contain the uncertain parameter $a_3(t)$, LMF and UMF are obtained as

$$\begin{aligned}\underline{\vartheta}_1(x_1(t)) &= 0.5\left(1 + \frac{0 - x_1(t)}{a_4}\right)\\ \bar{\vartheta}_2(x_1(t)) &= 1 - \underline{\vartheta}_1(x_1(t))\\ \bar{\vartheta}_1(x_1(t)) &= 0.5\left(1 + \frac{5 - x_1(t)}{a_4}\right)\\ \underline{\vartheta}_2(x_1(t)) &= 1 - \bar{\vartheta}_1(x_1(t)).\end{aligned}$$

In order to represent the varying of the uncertain parameter by the LUMFs, the weight coefficients of the first rule are supposed as $\bar{\beta}_1(t) = 0.5\sin^2(x_1(t))$ and $\underline{\beta}_1(t) = 1 - \bar{\beta}_1(t)$. Then, the MFs are determined by

$$\begin{aligned}\vartheta_1(x_1(t)) &= \underline{\vartheta}_1(x_1(t))\left(1 - \bar{\beta}_1(t)\right) + \bar{\vartheta}_1(x_1(t))\bar{\beta}_1(t)\\ \vartheta_2(x_1(t)) &= 1 - \vartheta_1(x_1(t)).\end{aligned}$$

The PDF of stochastic transmission delay is considered as

$$m(s) = -(10/h)^2 s e^{10s/h}, \quad s \in [-h,\ 0],\ h = 0.04s$$

which can be expressed as $m(\tau(t)) = (10/h)^2\tau(t)e^{10(-\tau(t))/h}$ for $\tau(t) \in [0,\ h]$ and $\int_0^h m(\tau(t))d\tau(t) = 1$. The stochastic delay and its normalized distribution histogram are depicted in Fig. 1. For $\kappa = 1$, to construct the vector $\mathbf{m}(s)$ satisfying property $\dot{\mathbf{m}}(s) = \mathcal{M}\mathbf{m}(s)$, another item $m_1(s) = -(10/h)e^{10s/h}$ is selected. Then, the vector $\mathbf{m}(s)$ and corresponding parameters are derived as

$$\mathbf{m}(s) = \begin{bmatrix}-(10/h)^2 s e^{10s/h}\\ -(10/h)e^{10s/h}\end{bmatrix}, \mathcal{M} = \begin{bmatrix}10/h & 10/h\\ 0 & 10/h\end{bmatrix}.$$

By choosing $\alpha = 1$, $\nu_1 = 0.85$, $\nu_2 = 0.9$ and the nonunity saturation level $\bar{u} = 10$, the fuzzy controller gains obtained by Theorem 2 and mathematical operation in Remark 4 are

$$\begin{aligned}K_1 &= \begin{bmatrix}8.2735 & 0.2828 & -7.4539\end{bmatrix}\\ K_2 &= \begin{bmatrix}8.2591 & 0.2814 & -7.9925\end{bmatrix}.\end{aligned}$$

In simulation, by setting $x(0) = [1; -1; 2]$, the responses of system state and the curve of saturated control signal are drawn in Figs. 2 and 3. These figures show that the designed

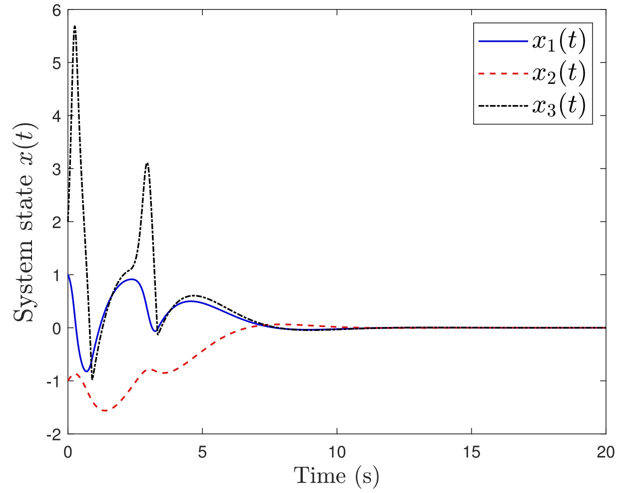

Fig. 2. State responses under our PDF-based distributed delay method for $h = 0.04$ s.

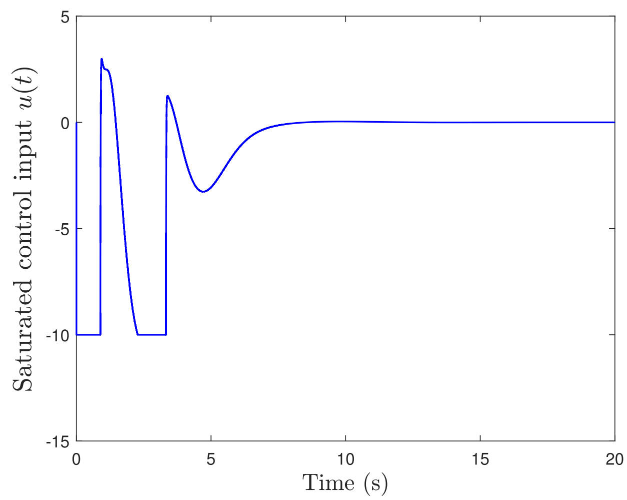

Fig. 3. Saturated control under our PDF-based distributed delay method for $h = 0.04$ s.

saturated controller is able to stabilize the system (55) with stochastic transmission delay and input saturation.

To demonstrate the merit of our method by introducing PDF in the delay model, the following comparison results with the existing ITV delay model $x(t - \tau(t))$ in [12] are carried out. By selecting the commonly used LKF $\mathcal{V}(t) = x^\top Px(t) + \int_{t-h}^{t}\Im(W, x(s))ds + \int_{-h}^{0}\int_{t-s}^{t}\Im(Y, \dot{x}(v))dvds$, and applying the Lyapunov method, the saturated fuzzy controller gains can be solved as

$$\begin{aligned}K_1 &= \begin{bmatrix}0.9751 & 0.1608 & -1.6517\end{bmatrix}\\ K_2 &= \begin{bmatrix}0.8675 & 0.1369 & -2.0087\end{bmatrix}.\end{aligned}$$

Then, the corresponding figures under the traditional ITV delay model are depicted in Figs. 4 and 5, respectively. By comparing Figs. 2 and 4, it is clear that our control method based on the delay model with PDF obtains better control performance than the existing approach.

In addition, the maximum allowable upper bound of stochastic delay under our PDF-based delay model and the conventional ITV delay model is listed in Table I, which tell

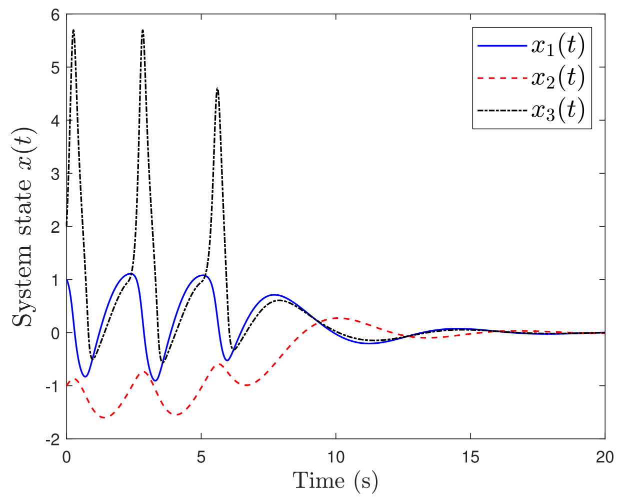

Fig. 4. State responses under the conventional ITV delay method for $h = 0.04$ s.

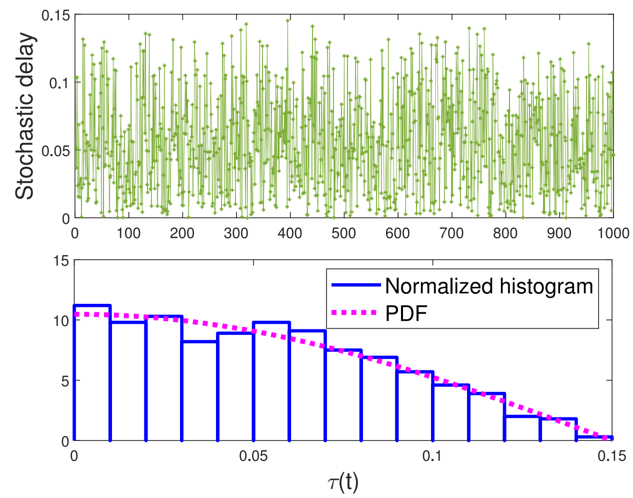

Fig. 6. Stochastic network delay and its PDF for $h = 0.15$ s.

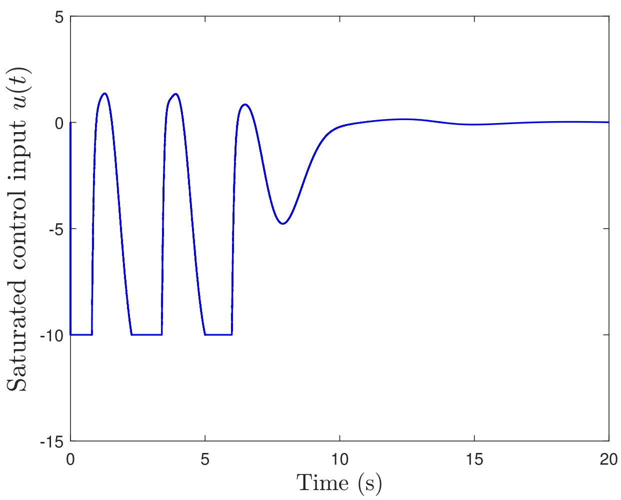

Fig. 5. Saturated control under the conventional ITV delay method for $h = 0.04$ s.

TABLE I
MAXIMUM ALLOWABLE UPPER BOUND OF DELAY $h_{\max}$

Method	$h_{\max}$(s)
Conventional ITV delay model in [12]: $x(t-\tau(t))$	0.046
PDF-based distributed delay model: $\int_{-h}^{0} m(s)x(t+s)ds$	0.130

that a larger allowable delay upper bound can be derived by our proposed method than the traditional method.

Thus, the above comparison results illustrate that the proposed PDF-based delay model is less conservative than the traditional ITV delay model.

Example 2: An IT2 T–S fuzzy system with the following parameters is considered:

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0.5 & -0.5 \end{bmatrix},\ B_1 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.5 & -1 \\ 0 & -1.5 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

The MFs are taken as

$$\vartheta_1(x_1(t)) = \frac{-1}{e^{-(x_1(t)+a(t))+1}} + 1$$
$$\vartheta_2(x_1(t)) = 1 - \vartheta_1(x_1(t))$$

where the uncertain parameter $a(t)$ belongs to $[0,\ 2]$. Then, the LUMFs are derived as

$$\underline{\vartheta}_1(x_1(t)) = \frac{-1}{e^{-x_1(t)}+1} + 1$$
$$\underline{\vartheta}_2(x_1(t)) = 1 - \bar{\vartheta}_1(x_1(t))$$
$$\bar{\vartheta}_1(x_1(t)) = \frac{-1}{e^{-(x_1(t)+2)}+1} + 1$$
$$\bar{\vartheta}_2(x_1(t)) = 1 - \underline{\vartheta}_1(x_1(t)).$$

The weight values of fuzzy rule 1 are assumed as

$$\bar{\beta}_1(t) = \sin^2(x_1(t)),\ \ \underline{\beta}_1(t) = 1 - \bar{\beta}_1(t)$$

which make the MFs as

$$\vartheta_1(x_1(t)) = \underline{\vartheta}_1(x_1(t))\big(1 - \bar{\beta}_1(t)\big) + \bar{\vartheta}_1(x_1(t))\bar{\beta}_1(t)$$
$$\vartheta_2(x_1(t)) = 1 - \vartheta_1(x_1(t)).$$

In this example, we consider the stochastic transmission delay, shown in Fig. 6, with the following PDF:

$$m(s) = \frac{\pi}{2h}\cos\Big(\frac{\pi s}{2h}\Big),\quad s \in [-h,\ 0]$$

which equals to $(\pi/2h)\cos([\pi\tau(t)]/2h)$ for $\tau(t) \in [0,\ h]$ and $\int_0^h m(\tau(t))d\tau(t) = 1$.

Following the similar way in Example 1, we have

$$\mathbf{m}(s) = \begin{bmatrix} \frac{\pi}{2h}\cos\big(\frac{\pi s}{2h}\big) \\ \frac{\pi}{2h}\sin\big(\frac{\pi s}{2h}\big) \end{bmatrix},\ \mathcal{M} = \begin{bmatrix} 0 & -\frac{\pi}{2h} \\ \frac{\pi}{2h} & 0 \end{bmatrix}.$$

By choosing $\epsilon = 3 \times 10^4$, $\alpha = 10$, $\alpha = 10$, $\nu_1 = \nu_2 = 0.9$ and the unity saturation level $\bar{u}_1 = \bar{u}_2 = 1$, the optimal estimation of DOA $\mathbb{A}_\delta$ for different h derived by our distributed-delay-dependent method solved by Problem 1 and the delay-independent method in [39] solved by Problem 2 are compared in Table II. Meanwhile, $\mathbb{A}_\delta$ for $h = 0.15$ is drawn in Fig. 7. From Table II and Fig. 7, one observes that a larger

TABLE II
COMPARISON OF THE ESTIMATION OF $\mathbb{A}_\delta$

$\mathbb{A}_\delta$	$h=0.1s$	$h=0.15s$	$h=0.2s$
Delay-independent method (49) in [39]	0.8873	0.6516	0.4096
Distributed-delay-dependent method (8)	0.9527	0.7241	0.4469

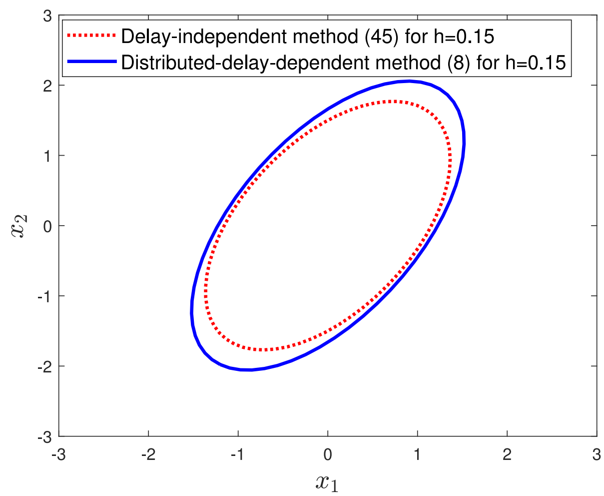

Fig. 7. Comparison of the estimation of $\mathbb{A}_\delta$ for $h = 0.15$ s.

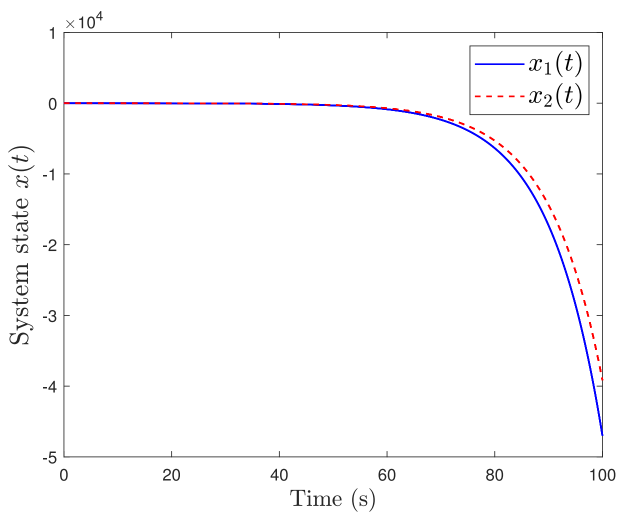

Fig. 8. State responses without control for $h = 0.15$ s.

DOA can be obtained by our distributed-delay dependent saturation representation method than the conventional method without considering the distributed delay term. This means that the distributed-delay-dependent term and auxiliary matrix F are helpful in reducing design conservatism.

The controller gains under $h = 0.15$ s are obtained as

$$K_1 = \begin{bmatrix} -0.5887 & 0.1155 \\ -0.1128 & -0.0850 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -1.3225 & 0.6157 \\ 0.5916 & -0.2610 \end{bmatrix}.$$

In simulation, under the initial condition $x(0) = [-2; 1]$, the state responses of system without control are shown in Fig. 8. Then, under the controller with the above parameters, the system state responses and the curve of the saturated control signal are illustrated in Figs. 9 and 10, respectively. From these figures, it is seen that the designed saturated controller is effective to guarantee the system stability when the stochastic transmission delay and input saturation happen.

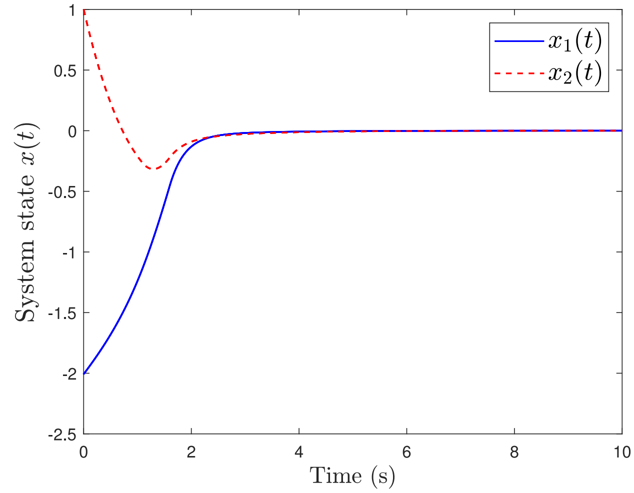

Fig. 9. State responses with saturated control for $h = 0.15$ s.

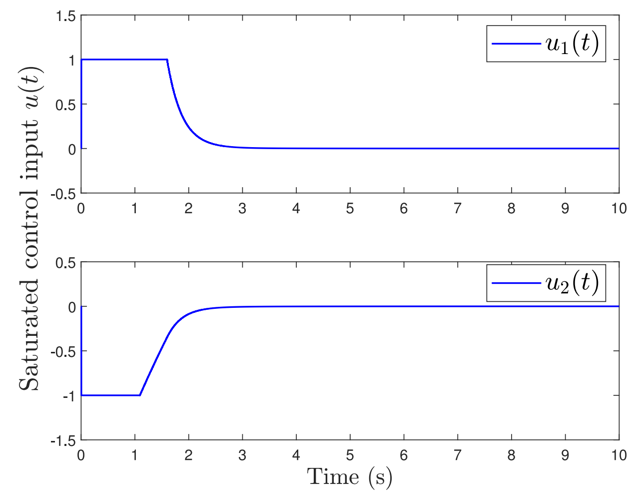

Fig. 10. Saturated control signal $u(t)$ for $h = 0.15$ s.

V. Conclusion

This article has investigated the saturated fuzzy controller design of networked IT2 T–S fuzzy systems with random delays. A more practical distributed delay model including the PDF is proposed to handle the stochastic network-induced delays. Moreover, an auxiliary distributed-delay-dependent vector is combined with the state vector to cope with the nonlinear saturation function. Then, sufficient less conservative conditions to design the saturated fuzzy controller are provided by using distributed-delay-dependent LKF and integral inequality. Finally, some simulation results are executed to demonstrate the merits of the developed approach. Note that the computation burden and complexity of the proposed method will increase exponentially along with the growth of the number of control channels and fuzzy rules. Then its application in multichannel and composite fuzzy systems may be limited. In addition, for some real networked systems, such as single-link robot, mass–spring–damping, and power systems, the security problem is an interesting and practical issue, which is not considered in this work. In the future, how to extend the distributed-delay-dependent saturated control method to design resilient saturated controller against cyber-attacks deserves further investigations.

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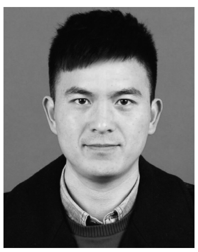

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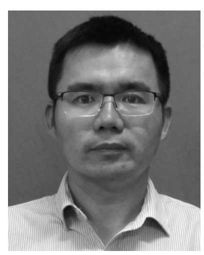

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